

The State of Composite Buckling in NX NASTRAN

Ben Names
Structures.Aero

For a stress analyst, there are typically two important factors when designing and sizing a structure. The first is the strength capability, captured by stress, and the second is the static stability, which is typically handled with a buckling analysis. We specialize in the analysis of composite structures, at Structures.Aero. I have come across a few oddities that generally only come up when running buckling on composite structures that I felt needed documenting. The summary of my findings are that as of version 12.2 of NX Nastran (release date September 2018), the best way to run a buckling analysis when using plate elements is to use CQUAD4/CTRIA3 elements with no offsets applied to the elements.

1.0 Buckling with offsets

The first issue I came across deals with element offsets, and although it isn't a problem exclusive to composites, it is more likely to be an issue for composite structures. When analyzing composite parts, analysts often receive CAD for the tool surface which corresponds to the OML of the part, and so naturally as analysts we mesh the surface and then offset the elements so that the bottom of the layup corresponds with the tool surface we got. Documented in the QRG under remark 8 for CQUAD4 and CQUAD8 elements, it is stated that the offset term (ZOFFS) should not be used when differential stiffness is calculated in solution 103 since "the differential stiffness calculation does not include the offset vectors." In fact, the preferred approach by some of our customers is to run the analysis using CQUADR elements, which were originally created to add a drilling DOF to the CQUAD4 element. This idea got me thinking about a simple buckling problem I was shown in my static stability course at school. This exercise showed that adding eccentric loading to a column (such as an element offset) doesn't change its buckling eigenvalue within the linear static domain. This can be illustrated by the following simple example. Take a simply supported column with the following dimensions:



Figure 1: Simply supported beam buckling with eccentricity

Writing out the Euler-Bernoulli beam equation, we get:

$$EI y'' = M = -P(y - e) \quad (1)$$

If we assume the deflection shape to take the form:

$$y = A \sin\left(\sqrt{\frac{P}{EI}} x\right) + B \cos\left(\sqrt{\frac{P}{EI}} x\right) - e \quad (2)$$

Being a simply supported beam, we know that:

$$\begin{aligned} y(0) &= 0 \\ y(L) &= 0 \end{aligned} \quad (3)$$

When you solve for A and B and substitute them back into equation 2, you get:

$$y = e \left[\tan\left(\sqrt{\frac{P L}{EI}}\right) \sin\left(\sqrt{\frac{P}{EI}} x\right) + \cos\left(\sqrt{\frac{P}{EI}} x\right) - 1 \right] \quad (4)$$

From the classical Euler column buckling, we know that $\lim_{x \rightarrow \pi/2} \tan(x) = \infty$, which is how we get:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad (5)$$

The more interesting thing to note is that the eccentricity e multiplies the entire displacement solution in equation 4, which means two things. Obviously, we have the trivial case that $\lim_{e \rightarrow \infty} y(x) = \infty$, but the more interesting idea to consider is that no matter the value of e , the displacement will diverge when the tangent term diverges. Put simply, the offset has no bearing on the actual buckling eigenvalue of a buckling column.

To test this idea with a FEM, I created six different rectangular plate models in which offsets and element types were varied:

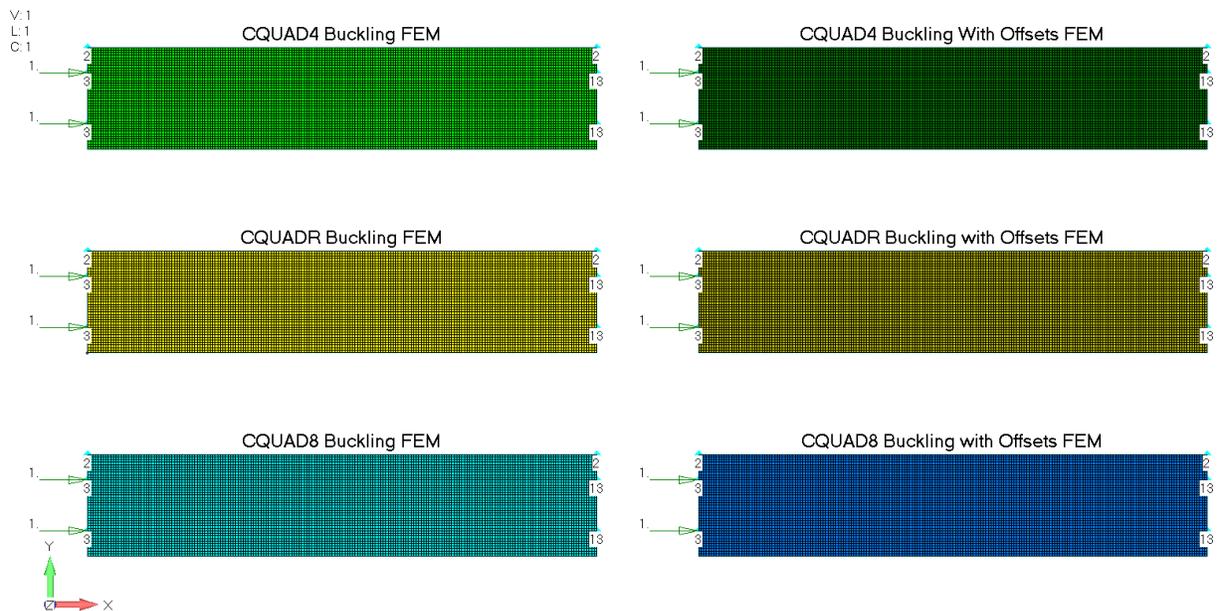


Figure 2: Metallic plate test fem buckling models with plate boundary conditions

Each plate has the dimensions of 1"x5" with a plate thickness of $t = 0.05$ ", and all plates are made of a generic Aluminum with $E = 10.3$ Msi.

To get a baseline, I first wanted to test these plates against a simple Euler-column buckling solution. To that end, a unit load was applied to the plate at one end, and each plate was simply supported. Using equation 5, we can calculate the $P_{cr}=42.4$ lbs:

Table 1: Analytical Euler column buckling results

Euler-Column Buckling Analytical Solution						
E	b	t	L	I	K	P_{cr}
10300000	1	0.05	5	1.04167E-05	1	42.4

The results from Nastran paint a clear picture, and quickly enumerate why using CQUADR's are the preferred element. All three element types produce almost exact results when no offsets are applied to the model, but only CQUADR elements correctly produce an unchanged eigenvalue when using offsets.

Table 2: Plate buckling results with Euler column boundary conditions

Model	Eigenvalue	Error
CQUAD4	42.5	0.4%
CQUAD4 OML Offset	3.3	92.3%
CQUADR	42.5	0.4%
CQUADR OML Offset	42.6	0.5%
CQUAD8	42.5	0.4%
CQUAD8 OML Offset	3.3	92.3%

To further explore buckling with offsets, I wanted to test more complex plate buckling with the premise that as with column buckling, linear plate buckling eigenvalues shouldn't be affected by adding offsets. Taking the same six plates, I created a buckling analysis which used the boundary conditions and load from the Euler Column problem above to calculate the internal compressive loads, and then a new pinned boundary condition to solve the buckling eigenvalue problem seen below.

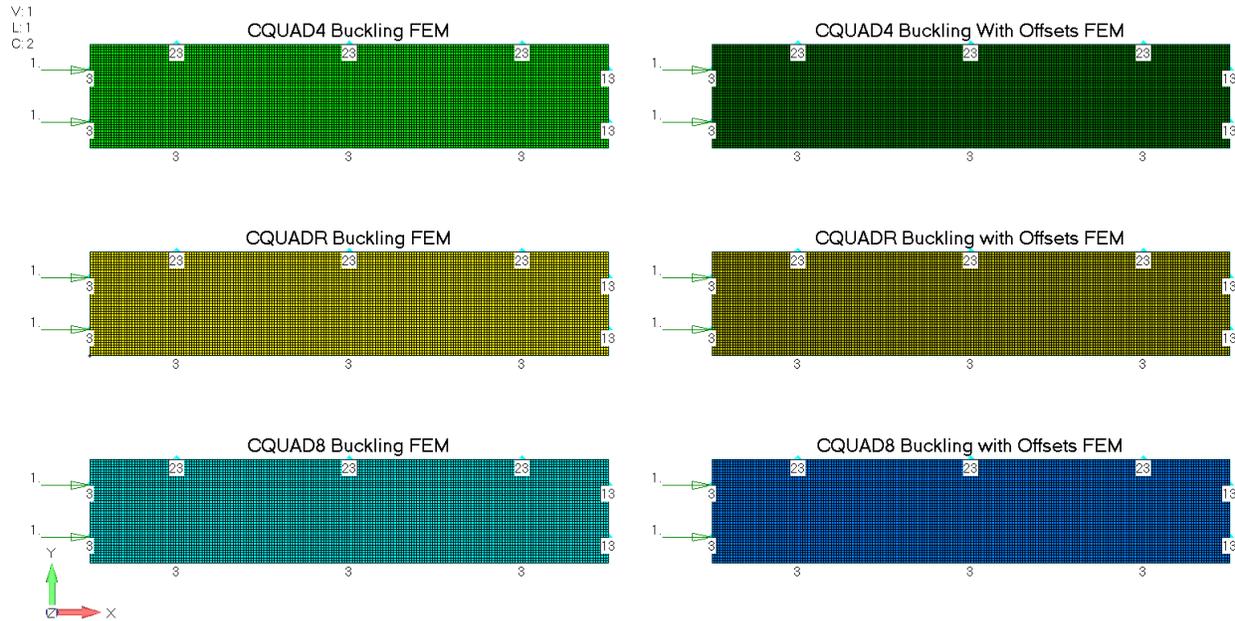


Figure 3: Metallic plate test fem buckling models with plate boundary conditions

The reason for using different boundary conditions between the static part of the solution and the eigenvalue part of the solution was to guarantee that for the plates with offsets, no y-compressive loads were introduced at the BC's due to in-plane and out-of-plane bending coupling. For example, when a panel is offset and an x-compressive load is applied, because of membrane/bending coupling the plate will also develop a non-negligible N_y plate loading.

```

TITLE = SS Plate Buckling
ECHO = NONE
DISPLACEMENT(PLOT) = ALL
SPCFORCE(PLOT) = ALL
FORCE(PLOT,CORNER) = ALL
SUBCASE 1
LOAD = 1
SPC = 1
SUBCASE 2
LABEL = BUCKLING CASE
METHOD = 1
SPC = 2
BEGIN BULK

```

Figure 4: Nastran case control statements for plate buckling

To analytically solve the problem, I used the following equation C5.1 from Bruhn in Chapter C5:

$$\sigma_{cr} = \frac{\pi^2 K_c E}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2 \quad (6)$$

For a simply-supported plate with an aspect ratio of 5, $K_c \approx 4$. To get the critical load, the critical stress can be multiplied by the cross-sectional area $b*t$, and shows that for this plate, the critical buckling load $P_{cr} = 4753$ lbs:

Table 3: Analytical plate buckling results

Simply Supported Uniaxial Plate Buckling Analytical Solution						
E	b	t	K	nu	sig	P _{cr}
10300000	1	0.05	4	0.33	95067	4753

Again, CQUAD4 and CQUAD8 elements fail to get the right buckling solution when offsets are applied, but the CQUAD8 plate without offsets isn't even able to capture the correct buckling solution. This result is a huge disappointment to me for two reasons. The first is that I have always been a firm believer in using higher order elements since they produce more accurate results for the same DOF, and clearly that isn't the case. The second is that clearly there is something wrong with the CQUAD8 element, likely in the geometric stiffness matrix. Until this bug is fixed, I couldn't recommend anyone use this element. Aside from that, this test produced somewhat similar results to that of the Column buckling with the CQUADR getting approximately the same eigenvalue with and without offsets, and the CQUAD4 only getting the correct eigenvalue with no offsets.

Table 4: Plate buckling results with simply supported boundary conditions

Model	Eigenvalue	Error
CQUAD4	4573	3.8%
CQUAD4 OML Offset	391	91.8%
CQUADR	4575	3.8%
CQUADR OML Offset	5499	15.7%
CQUAD8	4318	9.2%
CQUAD8 OML Offset	369	92.2%

As one other piece of commentary, for those who might be concerned with the ~4% error produced by the CQUAD4 and CQUADR elements with no offset, this is likely because in the y-direction, the b/t=20, and thick plate (transverse shear deformable effects) are likely coming into play, making the analytical Bruhn solution overly stiff as it assumes pure bending.

Finally, I wanted to compare the buckling solutions these plates produced under an applied shear load. An analytical solution (equation C5.4) for this problem can also be found in Bruhn Chapter C5:

$$\tau_{cr} = \frac{\pi^2 K_S E}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2 \quad (6)$$

Taking the same six plates, I created a buckling analysis which applied a unit line load around the perimeter of the plate intended to manifest as a unit N_{xy}. The plates were then constrained in the T3 direction around the perimeter, and pinned along one edge:

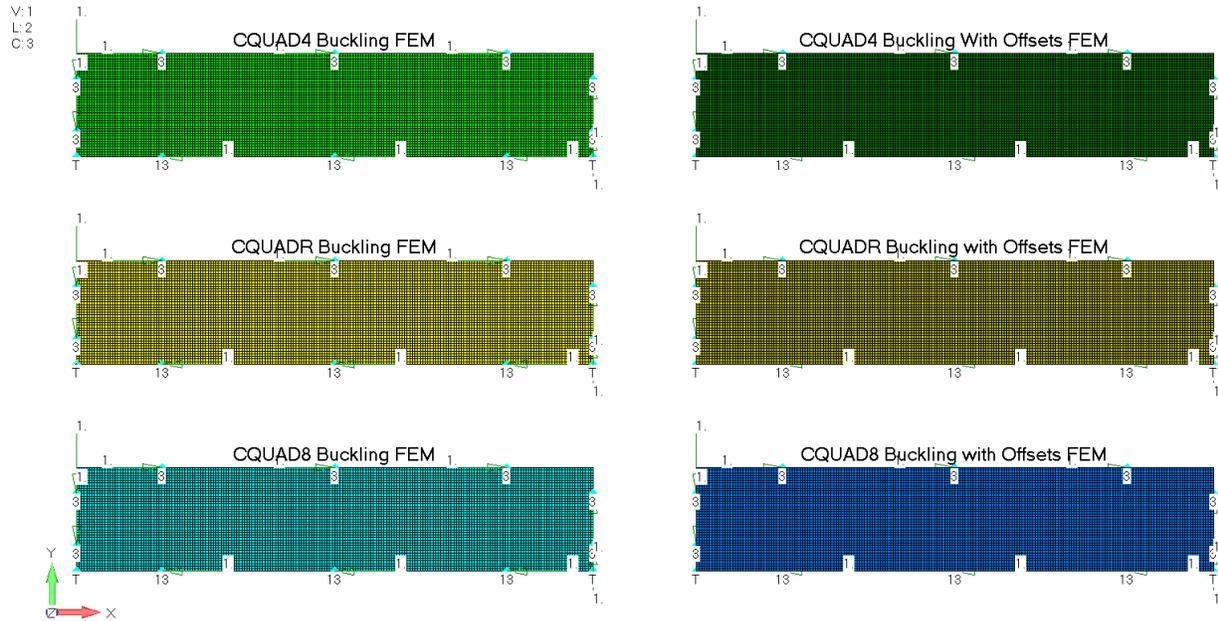


Figure 5: Metallic plate test fem buckling models with shear simply supported boundary conditions

For a simply-supported plate with an aspect ratio of 5, $K_c \approx 5.5$. To get the critical running load, the critical stress can be multiplied by the cross-sectional thickness t , and shows that for this plate, the critical buckling running load $P_{cr} = 52287 \text{ lb/in}$:

Table 5: Analytical plate shear buckling results

Simply Supported Shear Buckling Analytical Solution						
E	b	t	K	nu	sig	P_{cr}/L
10300000	1	0.05	5.5	0.33	130717	6536

Interestingly in this case, both CQUAD4 and CQUADR elements produce the correct buckling eigenvalues with and without offsets.

Table 6: Plate shear buckling results with simply supported boundary conditions

Model	Eigenvalue	Error
CQUAD4	6333	3.1%
CQUAD4 OML Offset	6279	3.9%
CQUADR	6337	3.0%
CQUADR OML Offset	6350	2.8%
CQUAD8	5240	19.8%
CQUAD8 OML Offset	1356	79.3%

It is worth noting though that CQUAD4 elements with offsets produce several “junk” solutions. Below is a comparison of the CQUADR buckling mode with offsets, along with several of the CQUAD4 buckling modes:

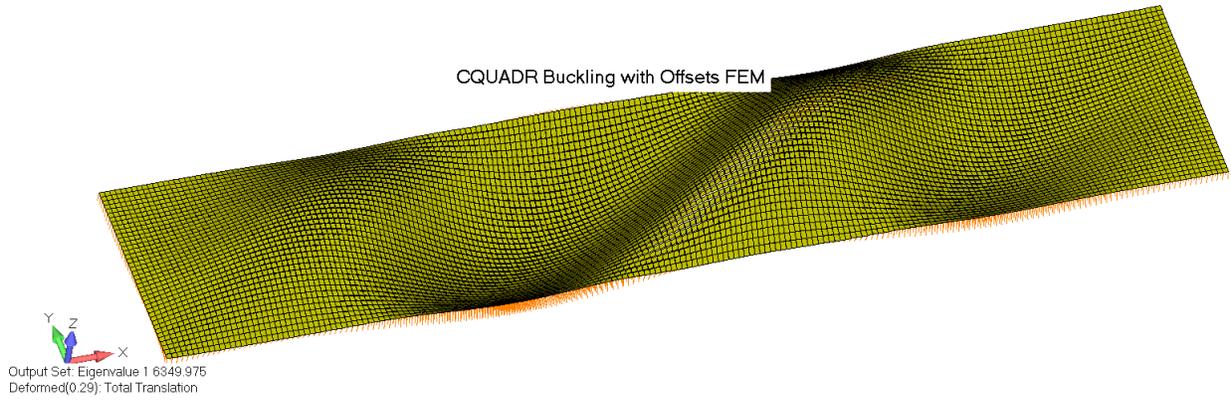


Figure 6: CQUADR first shear buckle mode with offsets

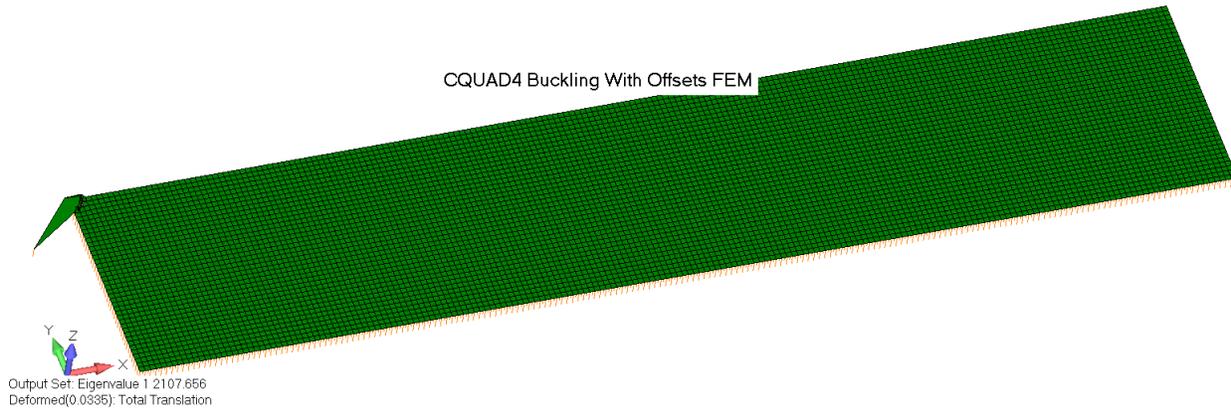


Figure 7: CQUAD4 first shear buckle mode with offsets

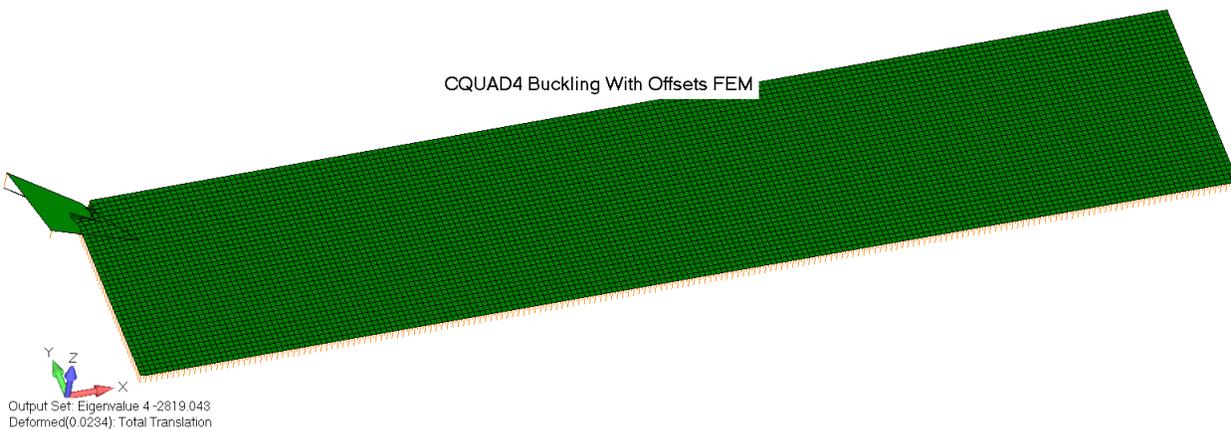


Figure 8: CQUAD4 fourth shear buckle mode with offsets

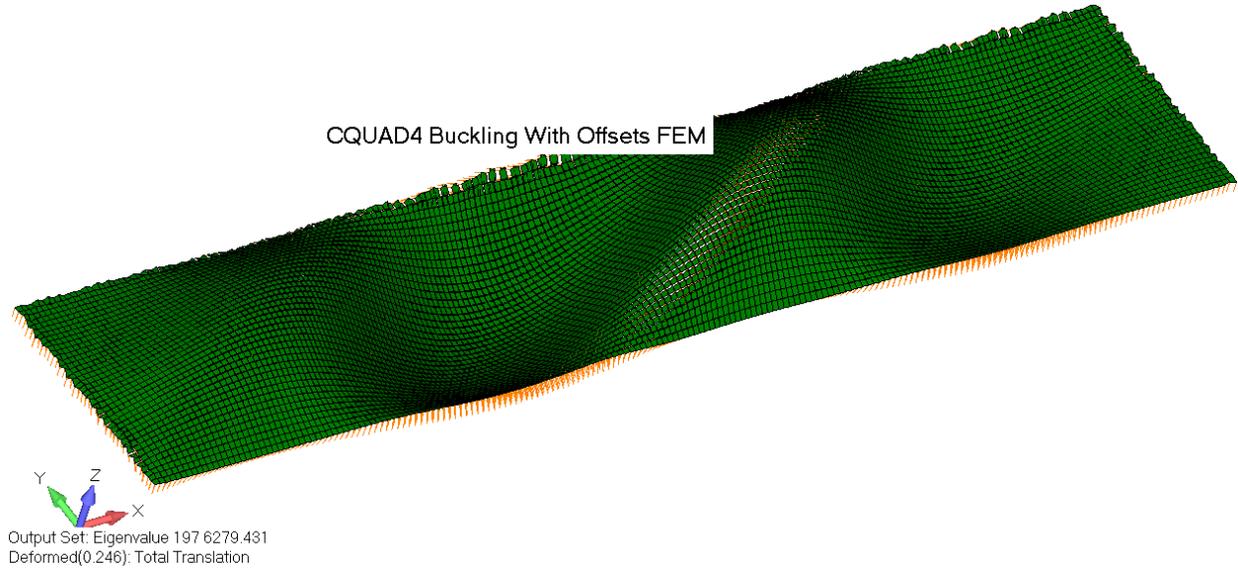


Figure 9: CQUAD4 197th shear buckle mode with offsets

Within the context of these three buckling problems, it would seem as though the only way to accurately analyze buckling with plate elements is to model a structure with CQUAD4 elements at the midplane, or model them with CQUADR elements.

2.0 Buckling with core

Another common feature within composite structures is the inclusion of core within a layup to increase a structure's bending stiffness EI and help it to resist buckling. Unfortunately, because core is very soft this can lead a panel to have a dramatically lower transverse shear stiffness. Over several different projects, many of our team members have noticed short wavelength buckles on cored panels, but we had no way to evaluate whether they were real, or what they might have meant. To explore the significance of the transverse shear stiffness on the buckling behavior of a structure, I returned to the simply supported plate which resulted in an Euler column buckling load of 338.9 lbs. The governing buckling differential equations for a shear deformable beam are:

$$\begin{aligned}
 -\frac{d}{dx} \left[GAK \left(\frac{dW}{dx} + \Psi \right) \right] + P \frac{d^2W}{dx^2} &= 0 \\
 -\frac{d}{dx} \left(EI \frac{d\Psi}{dx} \right) + GAK \left(\frac{dW}{dx} + \Psi \right) &= 0
 \end{aligned}
 \tag{7}$$

Using these equations from Reddy's Intro to FEA, we can get the element stiffness matrices for a shear deformable beam using a reduced integration element scheme to overcome shear locking:

$$[K]_e = \frac{GAK}{4h_e} \begin{bmatrix} 4 & -2h & -4 & -2h \\ -2h & h^2 + \alpha & 2h & h^2 - \alpha \\ -4 & 2h & 4 & 2h \\ -3h & h^2 - \alpha & 2h & h^2 + \alpha \end{bmatrix}
 \tag{8}$$

$$[G]_e = \frac{1}{h_e} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

where $\alpha = 4EI/GAK$ and h is the element length. As an aside, for those who might be unfamiliar GAK is the cross-sectional transverse shear stiffness of a beam, and only becomes important when shear deformation in a beam is not negligible.

These element matrices can then be assembled into a global matrix, and used to solve for the critical buckling eigenvalues. For a simple rectangular cross-section, the shear correction factor is $5/6$, and so for this cross-section we have that $GAK = \frac{5}{6}(0.1 * 1.0) * \frac{10.3e6}{2(1+.33)} = 322681$. If we use 20 beam elements to represent the simply supported plate, we get $P_{cr} = 42.5$ with a buckling shape seen below:

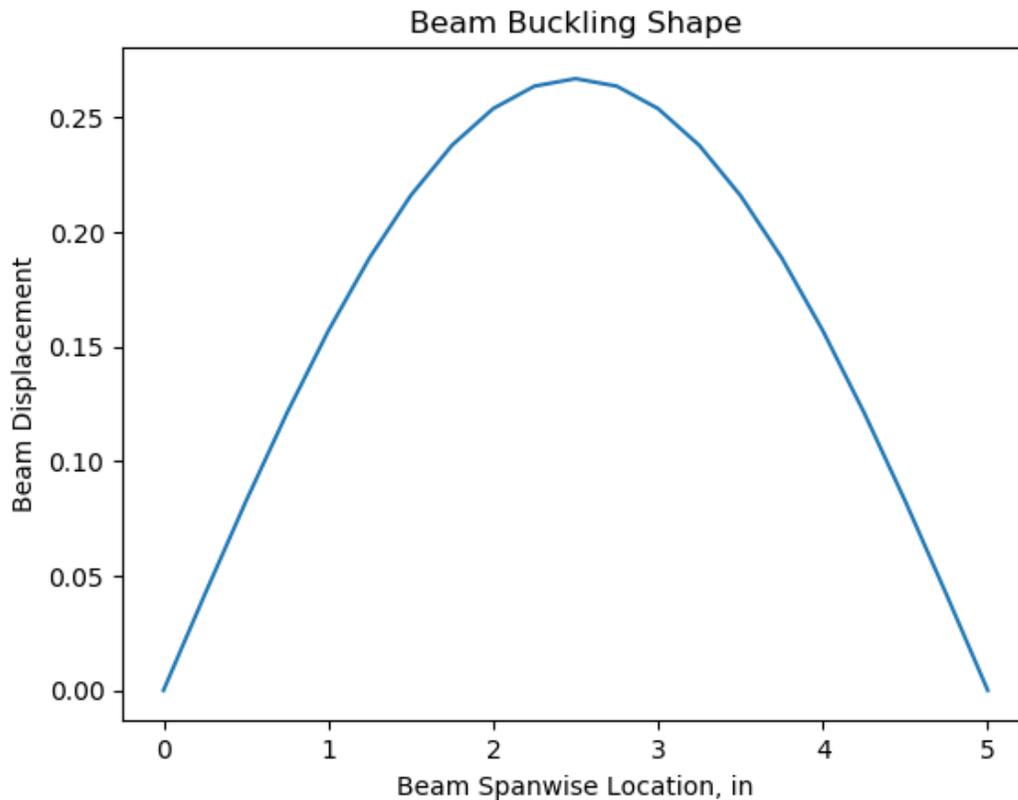


Figure 10: Timoshenko beam buckling mode shape

We can take two ideas away from these results. The first is that it is comforting to see the analytical Euler column buckling method, the CQUAD plate FEM, and the shear deformable beam FEM all yield nearly the exact same results. The second and more subtle observation we can make is that like one would expect, because the $L/t > 10$, we wouldn't expect shear deformable effects to be significant.

This begs the question, what happens to our buckling eigenvalue if we force our transverse shear stiffness to be lower? Below is a plot of the critical eigenvalue VS the transverse shear stiffness of the beam:

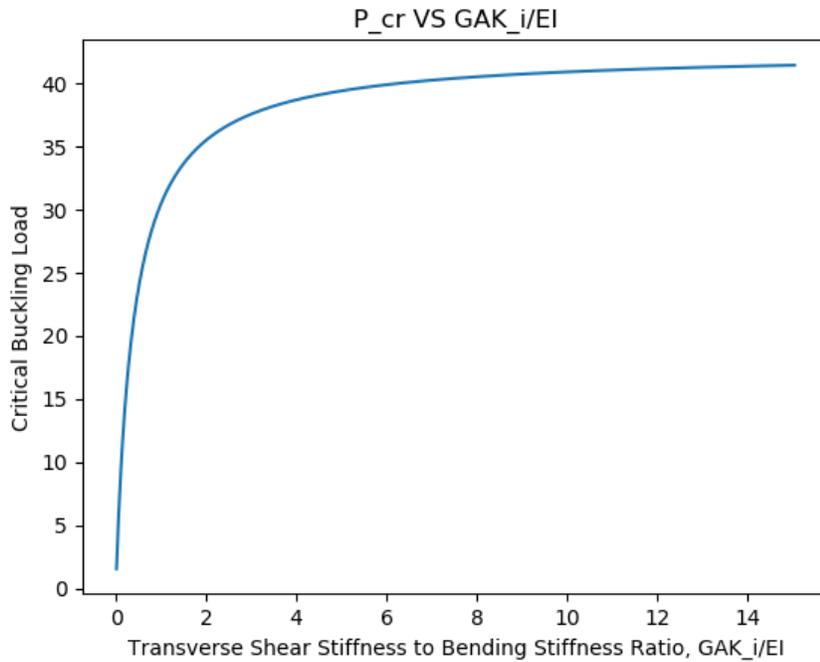


Figure 11: Timoshenko beam critical buckling eigenvalue vs transverse shear bending stiffness ratio

From this plot we can tell that changing GAK only slightly effects the critical buckling load until $GAK \cong 8EI$, at which point the critical eigenvalue drops dramatically. More interestingly, if we plot the buckling critical buckling load normalized by the transverse shear stiffness, we can see that

$$\lim_{GAK \rightarrow 0} P_{cr} = GAK:$$

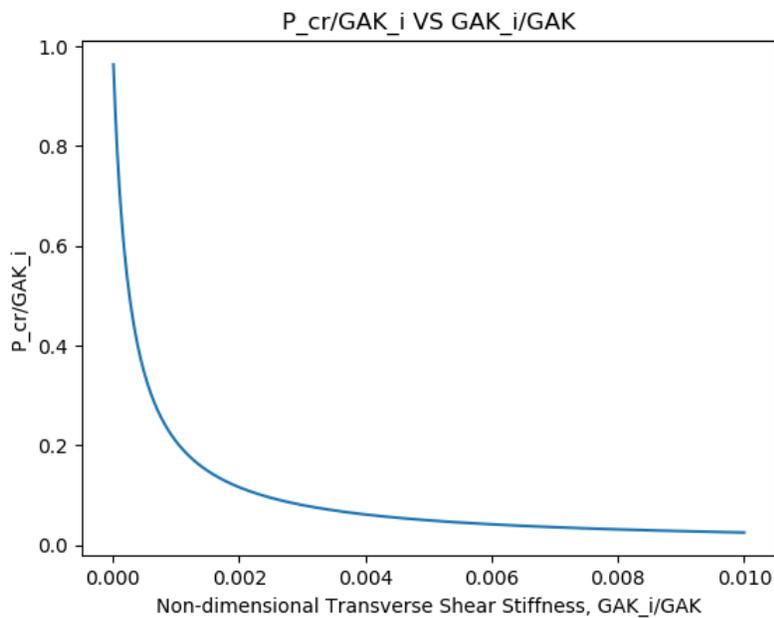


Figure 12: Timoshenko beam critical buckling eigenvalue vs normalized transverse shear stiffness

This makes it clear that regardless for how slender a beam is, if the transverse shear stiffness is close in magnitude to the bending stiffness, the buckling eigenvalue will be substantially influenced by both stiffness terms. Additionally, when $GAK \ll EI$, we get two interesting results. First, we find that $P_{cr} = GAK$. Second, we find that all of the eigenvalues approximately equal the transverse shear stiffness as illustrated by the figure below:

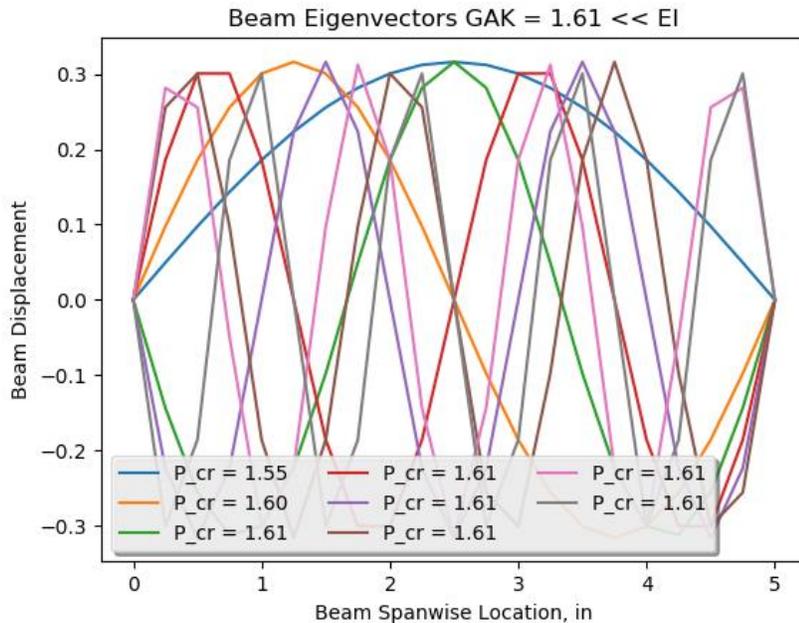


Figure 13: Timoshenko critical buckling mode shapes for beam with soft transverse shear stiffness

The implication here is that if a portion of a structure has a GAK which is very small, a buckling analysis will produce eigenvalue/eigenvector pairs for all degrees of freedom of the soft structure all at approximately the same eigenvalue. This is likely why cored panels exhibit so many short wavelength buckles, all at about the same eigenvalue.

Anecdotally, I had noticed that even when no offsets are applied to the elements, CQUAD4 and CQUADR elements produce significantly different eigenvalues when they are representing cored panels. To explore why this might be the case for cored panels, consider the following 4 thick plates which have overall dimensions of 1.0"x0.5" and are intended to highlight transverse shear effects:

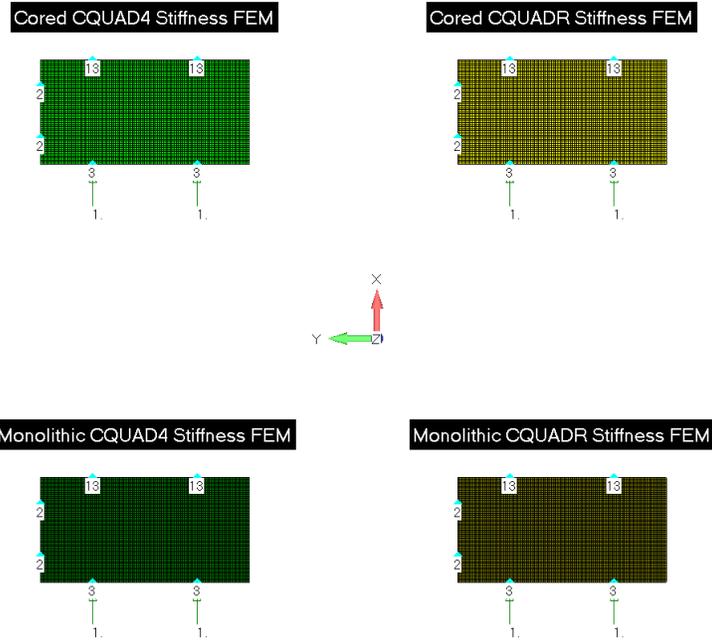


Figure 14: Composite test fems varying core

The layup for the bottom two plates is $[0/45_2/0_2/45_2/0_2/45_2/0]_s$, where all plies are representative graphite fabric with a 0.008" ply thickness and the following material properties:

Define Material - 2D ORTHOTROPIC

ID 3 Title Graphite Fabric Color 104 Palette... Layer 1 Type...

General Function References Nonlinear Creep Electrical/Optical Phase

Stiffness (E)

1	12160233.
2	12160233.

Shear (G)

12	638166.
1z	551143.5
2z	551143.5

Poisson Ratio (nu)

12	0.0359175
----	-----------

Limit Stress/Strain

Stress Limits Strain Limits

Dir 1 Dir 2

Tension	0.	0.
Compression	0.	0.
Shear	0.	

Specific Heat, Cp 0.

Mass Density 0.1

Damping, 2C/Co 0.

Reference Temp 0.

Tsai-Wu Interaction 0.

Thermal Expansion (A)

1	0.
2	0.

Thermal Conductivity (k)

0.	0.	0.
symmetric	0.	0.
		0.

fy Load... Save... Copy... OK Cancel

The layup for the top row of cored panels is [0/45_2/0/core/0/45_2/0] where the core is a Hexcel 1/8" cell size 3pcf core with a thickness of 1/8". The other plies in the cored layups use the graphite fabric from above.

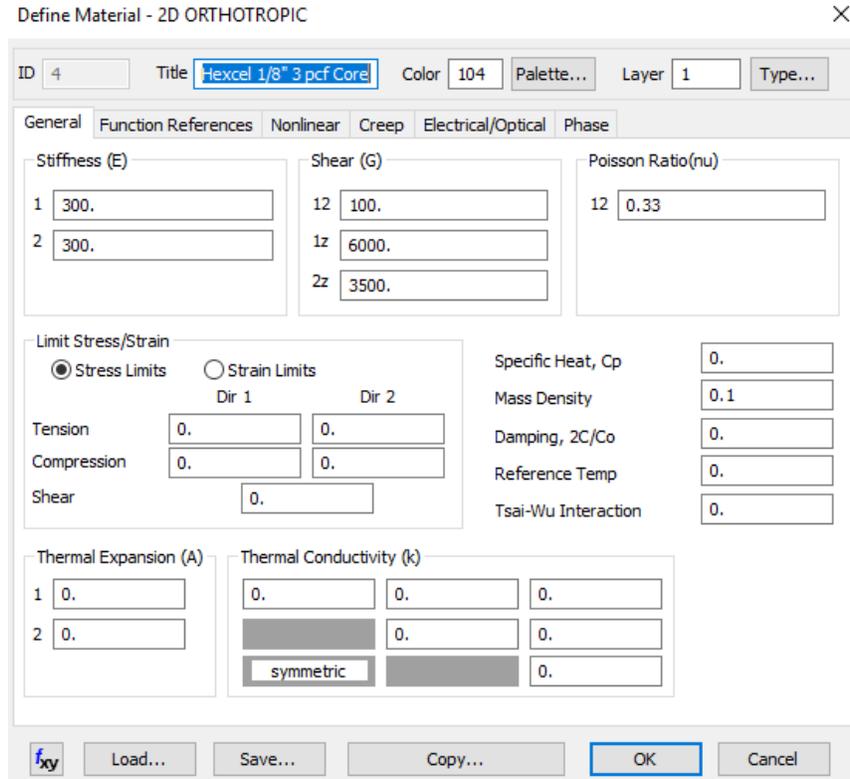


Figure 15: Sample core material properties

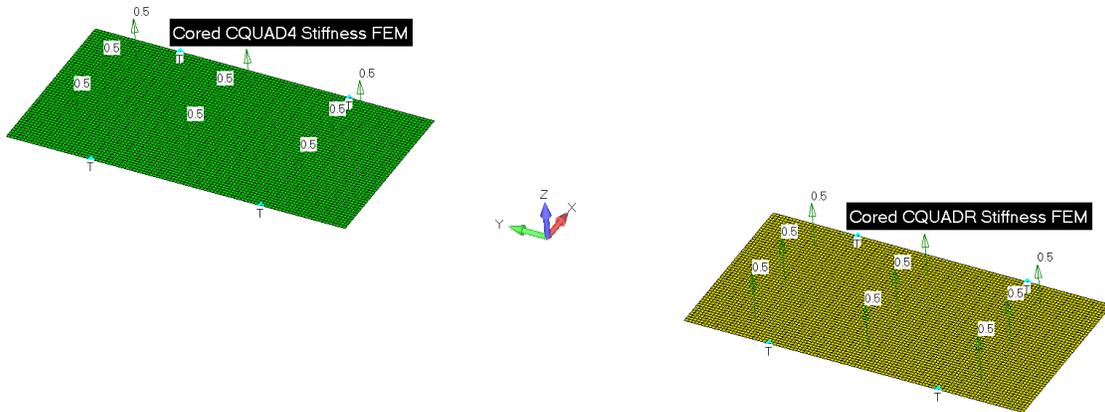
The last thing of note is that a unit compressive load is applied to each plate, and they are given the same pinned constraints as the metallic plates above. From the previous analysis, it should be no surprise that with no offsets, the monolithic CQUAD4 and CQUADR panels produce nearly the exact same buckling eigenvalue and mode shape. More interestingly though, the cored CQUAD4 and CQUADR panels produce dramatically different eigenvalues:

Table 7: Plate critical buckling eigenvalue using euler column boundary conditions

	CQUAD4	CQUADR
Monolithic Panel	62126 lbs	62131 lbs
Cored Panel	1210 lbs	709 lbs

From this we can clearly tell that something is wrong with either the CQUAD4 or CQUADR result for the cored panel, though it is unclear which is correct. From the shear deformable beam example, we found that the having accurate bending and transverse shear stiffnesses can be important to solving certain buckling problems. As such, the logical first step would be to make sure that the bending and transverse shear stiffnesses of the cored panels are correct. To do this, two approaches were taken. In the first approach the cored panels were loaded using a uniform 0.5 psi load distribution and were given a simply supported beam boundary condition.

V:1
L:8
C:4



Using an analytical solution out of Reddy's Intro to FEA, the deflection and rotation of a shear deformable beam can be expressed with:

$$w(\bar{x}) = \frac{q_0 L^4}{24 EI} (\bar{x} - 2 \bar{x}^3 + \bar{x}^4) + \frac{q_0 L^2}{2 GAK} (\bar{x} - \bar{x}^3) \quad (10)$$

$$\Psi(\bar{x}) = -\frac{q_0 L^3}{24 EI} (1 - 6 \bar{x}^2 + 4 \bar{x}^3)$$

where $\bar{x} = x/L$. These equations can then be arranged and solved for EI and GAK given the inputs of q_0 , L , and \bar{x} , $w(\bar{x})$ and $\Psi(\bar{x})$. The second approach is to use a code I wrote called AeroComBAT (a cross-sectional analysis tool) to solve for EI and GAK of this plate/beam separately. From AeroComBAT, the stiffnesses reported are:

Table 8: AeroComBAT generated cross-sectional stiffness values

	AeroComBAT w/ Core
GAKs	1225
EI	3770

By reading deflections and rotations from the FEM, the stiffnesses reported are:

Table 9: FEM derived cross-sectional stiffness values

Model	NID	L	xbar	T3	R2	EI	EI Error	GAKs	GAKs Error
CQUAD4	96846	0.5	0.02	2.02E-06	-1.51E-06	3450	-8.5%	1246	1.7%
CQUADR	107148	0.5	0.02	3.45E-06	-1.51E-06	3450	-8.5%	728	-40.6%

By comparing the results between what AeroComBAT produced and what the NASTRAN results produced, it is clear that when core (or a material with a comparatively low transverse shear stiffness) is

included in a layup, CQUADR elements incorrectly calculate the transverse shear stiffness of the stack. What this likely means is not that there is anything special with core, so much as when mixing materials in a stackup, CQUADR elements incorrectly calculate the transverse shear stiffness. This inaccuracy would be negligible if all materials within a layup have the same or approximately the same transverse shear stiffness. I would guess that the transverse shear stiffness of spar caps are also incorrect when modeled with CQUADR elements, as they would likely bias the transverse shear stiffness towards the softest plies in the cap stack.

One of the bigger takeaways from the shear deformable beam exercise above is that when GAK is very small compared with EI , $P_{cr} = GAK$ which is exactly what we find in this sample problem. The buckling eigenvalues for each cored plate almost exactly match the transverse shear stiffnesses of calculated for each panel. This also leads to a common misconception that when buckles (and in particular short wavelength buckles) appear in cored panels, this is an indication of a Shear Crimping issue, which is a short wavelength core instability whose critical running load can be calculated using:

$$N_{cr} = G t_{core} \quad (11)$$

where G is the transverse shear modulus of the core, and t_{core} is the thickness of the core. For the above plates, the critical shear crimping load $N_{cr} = 6000(0.125) = 750 \text{ lb/in}$, which in this case is close to the eigenvalue calculated when using CQUADR elements. This is likely purely a coincidence in that the transverse shear stiffness of CQUADR elements is incorrectly heavily biased towards the softest material in the stack. In short, no connection can be reliably drawn between the short wavelength buckles NX Nastran will produce and core crimping.

3.0 Summary

The findings of this paper are a little disheartening for any stress engineer that analyzes composite structures in NX Nastran. From the first section, it is clear that only CQUADR elements handle offsets correctly. However the second section shows that CQUADR elements have an incorrect transverse shear stiffness, so the only accurate way to run a buckling analysis on composite structures is to do so using CQUAD4 elements with no offsets. Though the results also shed a lot of light on some subtleties of buckling, it doesn't answer the question as to whether the short wavelength buckles produced by NX Nastran on cored panels are real, and if they are what their significance is. Often when these short wavelength buckles appear, they are also very mesh dependent. For example, if one were to see these buckles on a short wavelength panel, and then refine the mesh by doubling the nodes, one would see a similar mode shape with double the number of peaks in that mode at approximately the same eigenvalue. This mesh dependent oscillating solution that does not converge is a very typical sign of numerical convergence issue. It is my recommendation that these eigenvalues be ignored, though it would be better to rule these out by addressing the validity by running a nonlinear analysis.

4.0 Future work

Should there be an opportunity to investigate the short wavelength buckles in the future, the goal of the future study would be to determine the validity of the short wavelength buckles using nonlinear analysis, and possibly models using 3D core, as well as outlining workarounds to get accurate buckling solutions without having to sacrifice the presence of offsets within the model.