Femap Tutorial:
Shock Response Spectrum Analysis

IN THIS TUTORIAL:

• Background of Shock Response Spectrum Analysis
• Example FEA Models, Analytic Derivations
• Introduction to Direct Transient analysis

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COPV Model - Motivation

- Objective: Analyze mounting structure and the reaction forces that result from random vibration and shock analysis.
## Types of Dynamic Analysis?

<table>
<thead>
<tr>
<th>Type</th>
<th>Load Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modal/Eigenvalue</td>
<td>• None</td>
<td>• Undamped free vibrations</td>
</tr>
<tr>
<td></td>
<td>• Boundary conditions only</td>
<td>• Natural frequencies and mode shapes</td>
</tr>
<tr>
<td>Linear Frequency Response</td>
<td>• Loads varying as a function of frequency (Deterministic)</td>
<td>• Response at each frequency</td>
</tr>
<tr>
<td></td>
<td>• e.g. Sinusoidal/harmonic excitation</td>
<td></td>
</tr>
<tr>
<td>Linear Transient Response</td>
<td>• Time varying loads (Deterministic)</td>
<td>• Time varying response</td>
</tr>
<tr>
<td>“Shock” Response Spectrum</td>
<td>• Load spectrum across a range of frequencies representing shock environments (Deterministic)</td>
<td>• Approximate maximum response of model</td>
</tr>
<tr>
<td>Vibro-acoustics and Random Vibration</td>
<td>• Load spectrum representing the probability distribution of excitations across range of frequencies (Non-Deterministic)</td>
<td>• Probability distribution response functions of the model.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• RMS response values (i.e. response of the model to occur within a range of probabilities)</td>
</tr>
</tbody>
</table>
Why Spectral Analysis in the Frequency Domain?

- A long (many time points) transient analysis of a large (many DOF) model provides the most accurate solution.
- This can be computationally expensive.
- Sometimes a quick approximation of the maximum response is preferred.

Applications:
- Random response spectrum analysis is for non-deterministic vibrations
  - Parts on a manufacturing line
  - Airplanes flying or taxiing
  - Building subjected to earthquake
  - Spacecraft and Rocket sub-structure during launch
- Shock response spectrum analysis is for deterministic vibrations
  - Electrical/Optical component subjected to shock load
  - Impulsive load due to stage separation
  - Building subjected to seismic load

- Transient analysis should be used when nonlinearities are critical and cannot be ignored.
FEA Tutorial Model

- Approximate model of the full tank assembly:

  LENGTH = 28.87"
  MASS: 141.2977 lbm = 0.365971 slinches
  INTERTIA: \( I_{xx} = I_{yy} = \frac{mL^2}{2} = 25.4 \) slinch in\(^2\)

RBE2, Mass is independent node

No pressure preload in this model. Eliminate torsion with a DOF6 constraint on the mass node.
Normal Modes Analysis: Tank Axial Mode

\[ \sum F = 0 \]
\[ m \ddot{x} + (k_1 + k_2)x = 0 \]

Eigenvalue Problem: \[ (k_1 + k_2) - \omega^2 m) \lambda = 0 \]

\[ \omega = \sqrt{\frac{k_1 + k_2}{m}} \]
\[ f = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}} \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{150001 \text{ lb} \text{ in}}{141.2977 \text{ lb}}} = 101.8929 \text{ Hz} \]

Output Set: Mode 1, 101.8929 Hz, Deformed(1.653): Total Translation

T3 Mass Participation
Normal Modes Analysis: Tank Lateral and Vertical Modes

\[ l = \frac{mL^2}{2}, \quad u_1 = u_c - \frac{\theta L}{2}, \quad u_2 = u_c + \frac{\theta L}{2} \]

\[ \sum F_c = 0, \quad \sum M_c = 0 \]

EOM:

\[ \begin{align*}
\Sigma F_c: & \quad m\ddot{u}_c + k_1 \left( u_c - \frac{\theta L}{2} \right) + k_2 \left( u_c + \frac{\theta L}{2} \right) = 0 \\
\Sigma M_c: & \quad l\ddot{\theta} + \frac{k_1 L}{2} \left( u_c - \frac{\theta L}{2} \right) + \frac{k_2 L}{2} \left( u_c + \frac{\theta L}{2} \right) = 0 \\
& \quad l\ddot{\theta} + \frac{L}{2} (k_2 - k_1) u_c + \frac{L^2}{4} (k_1 + k_2) \theta = 0
\end{align*} \]

Eigenvalue Problem:

\[ [K - \omega^2 M] \lambda = 0 \]

\[ \begin{vmatrix}
(k_1 + k_2) - \omega^2 m & \frac{L}{2} (k_2 - k_1) \\
\frac{L}{2} (k_2 - k_1) & \frac{L^2}{4} (k_1 + k_2) - \omega^2 l
\end{vmatrix} = 0 \]

Quadratic Formula:

\[ \omega^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad f = \frac{1}{2\pi \sqrt{\omega^2}} \]

\[ k1_{\text{Lateral}} = 75000 \]
\[ k2_{\text{Lateral}} = 85000 \]
\[ k1_{\text{Vertical}} = 350000 \]
\[ k2_{\text{Vertical}} = 90000 \]

\[ f_{\text{Lateral}} = 104.93 \text{ Hz, } 182.45 \text{ Hz} \]
\[ f_{\text{Vertical}} = 131.65 \text{ Hz, } 323.24 \text{ Hz} \]
Normal Modes Analysis: Tank Lateral and Vertical Modes

Lateral Modes
- Output Set: Mode 2, 104.9263 Hz, Deformed(1.805): Total Translation
- Output Set: Mode 4, 182.4483 Hz, Deformed(2.948): Total Translation

Vertical Modes
- Output Set: Mode 3, 131.6471 Hz, Deformed(2.634): Total Translation
- Output Set: Mode 5, 323.2424 Hz, Deformed(3.28): Total Translation
Normal Modes Analysis: Tank Lateral and Vertical Modes

T1 Mass Participation

T2 Mass Participation

Effective modal mass for mass normalized orthogonal eigenvectors:

\[ m_{eff\,ij} = L_{ij}^2, \quad \text{where} \quad [L] = \phi^T [M] \]

For the lateral and vertical directions, which have two modes each:

\[ m_{eff\,ij} = \begin{bmatrix} (\phi_{11} m)^2 & (\phi_{21} l)^2 \\ (\phi_{12} m)^2 & (\phi_{22} l)^2 \end{bmatrix} \]

If \( k_1 = k_2 \), then \( \phi_{12} = \phi_{21} = 0 \)

Therefore, effective modal mass of the translational and rotational modes is decoupled.

This will be key later on.
12.4 Response Spectrum Analysis

Response spectrum analysis is an approximate method of computing the peak response of a transient excitation applied to a simple structure or component. This method is used in civil engineering to predict the peak response of a component on a building that is subjected to earthquake excitation; it is also used in aerospace engineering to predict the peak response of equipment in a spacecraft that is subjected to an impulsive load due to stage separation. Because it is an approximate method, response spectrum analysis is often used as a design tool. Response spectrum analysis is also called shock spectrum analysis.

There are two parts to response spectrum analysis: (1) generation of the spectrum and (2) use of the spectrum for dynamic response such as stress analysis. Both are available in NX Nastran.
Shock Response Spectrum Analysis (cont.)

Note that the peak response for one oscillator does not necessarily occur at the same time as the peak response for another oscillator. Also note that there is no phase information since only the magnitude of peak response is computed. It is assumed in this process that each oscillator mass is very small relative to the base structural mass so that the oscillator does not influence the dynamic behavior of the base structure.

Once a spectrum is computed, it can be used for the dynamic response analysis of an NX Nastran model of the component. For example, the spectrum generated for a floor in a building that is subjected to an earthquake can then be applied to a complex model of a piece of equipment attached to that floor. The peak response of each mode of the equipment model is obtained from the spectrum, and these peak modal responses are combined to create the overall response.

Because the peak responses do not all occur at the same time and only the magnitude of peak responses are computed, various methods are used to combine the peak responses into the overall response. The combination methods implemented in NX Nastran are SRSS (square root of the sum of the squares), ABS (absolute values), and NRL (U.S. Navy shock design modal summation). The typical response quantities computed are grid point displacements and element stresses.

Figure 12-1. Response Spectrum Generation
Shock Response Spectrum Analysis (cont.)

- Shock spectra for 5% damping was provided.
- Typically will have multiple spectrum at various levels of damping.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Qualification level (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>28</td>
</tr>
<tr>
<td>1850</td>
<td>5090</td>
</tr>
<tr>
<td>10000</td>
<td>5090</td>
</tr>
</tbody>
</table>

Table 3-7: Shock environment level
Create Ground Mass and Constraints

- Shock analysis will be performed in x-, y-, and z- directions
- Single point excitation:
  - Create RBE2 that connects to previous constraint nodes.
  - Create CONM2 on the independent node of this RBE.
  - The CONM2 should have a mass of at least 1e6 times system mass. This is simulating the ground.

For X-direction excitation, create two constraint sets. A 23456 constraint on the ground mass, and a 1 constraint on the ground mass.
Create Spectrum and Damping Functions

Be sure to read the slide in the Appendix for a very important note on this function definition.

- **Create 3 Functions:**
  - Spectrum: Acceleration vs. Freq; define one function for each spectra. In this case, only one spectra is used.
  - Damping Table: If multiple spectrum are used, this function assigns a damping value to each spectra. This must be defined even if only one spectra is used.
  - Damping Function: Damping coefficient vs. Frequency; Can be any value, the spectrum load level is interpolated based on the damping table.
Define SRS Modal Analysis

- Set spectrum type: Acceleration
- Set damping table: References acceleration spectrum
- Set combination method: SRSS
- Set SUPORT constraint set: This is constraint on DOF being excited
- Set damping function

If many modes, setting to finite number can help reduce run time. Just make sure enough modes are included to capture sufficient majority of mass participation.
Shock Response Spectrum Analysis Results

- Result is a single output set containing the combined responses of the spectrum analysis.
- The results from the x-, y-, and z-direction analyses are tabulated below.
- Lateral loads are far too high for mounting structure to survive.
- Mass participation of second vertical mode is driving loads up.
- Redesign mounting structure to have lower and more equivalent stiffness in the vertical direction.

<table>
<thead>
<tr>
<th>Direction</th>
<th>-Z End Reactions (lbs)</th>
<th>+Z End Reactions (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X (Lateral)</td>
<td>2222</td>
<td>2091</td>
</tr>
<tr>
<td>Y (Vertical)</td>
<td>6087</td>
<td>3526</td>
</tr>
<tr>
<td>Z (Axial)</td>
<td>0</td>
<td>4090</td>
</tr>
</tbody>
</table>

85% mass participation at 131.6 Hz, and 15% mass participation at 323.2 Hz. Interpolating shock spectrum, the g-level at those frequencies are 45.7 g’s and 226.8 g’s, respectively.

Approximate Reaction (ABS Combination Method):
\[
F_y = m \left(0.85(45.7) + 0.15(226.8)\right)
\]
\[
F_y = 10294 \text{ lbs}
\]
Shock Response Spectrum Analysis Results

- Adjust model to reduce reactions in the vertical direction:
  - DOF2 stiffness of –Z CBUSH changed from 350000 to 85000 lb/in.
  - The vertical modes drop to 108 Hz and 188 Hz.
  - Mass participation of second mode is now insignificant.
  - The reaction forces for y-direction excitation were reduced to 2378 lbs and 2311 lbs at the –Z and +Z ends of the tank, respectively.

<table>
<thead>
<tr>
<th>Direction</th>
<th>-Z End Reactions (lbs)</th>
<th>+Z End Reactions (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X (Lateral)</td>
<td>2222</td>
<td>2091</td>
</tr>
<tr>
<td>Y (Vertical)</td>
<td>2378</td>
<td>2311</td>
</tr>
<tr>
<td>Z (Axial)</td>
<td>0</td>
<td>4090</td>
</tr>
</tbody>
</table>
Direct Transient Analysis

• Perform transient analysis of half-sine shock wave at critical frequency in the vertical direction (i.e., 108.2 Hz).

• The dynamic amplification factor (DAF) of each mode was accounted for when the shock spectrum was derived.

• For the direct transient analysis to be consistent with the spectral analysis we have already performed in this tutorial, we must first determine the DAF for this particular mode.

• We will do this by performing a transient analysis of a half-sine unit step at the 108.2 Hz frequency.
Define the Half-Sine Shock Wave

- Create a “vs. Time” function:
  - Select function type 1 (vs. Time)
  - Select “Equation” radio button
  - Delta X = 0.5/108.1918/101
  - X = 0 and To X = 0.5/108.1918
  - Y = sin(360*108.1918!*x)
  - Click “Add” button
  - Select “Single Value” radio button
  - X = .05
  - Y = 0.0
  - Click “Add” button
  - Click “OK” button
Define Enforced Displacement Load

- Create an enforced displacement load set on the node of the ground mass.
- Be sure to reference the shock wave function just created.
- Create a fixed constraint on the node of the ground mass.
Create Direct Transient Analysis Set

2X damping factor

Critical Frequency
Direct Transient Analysis Results

• Chart the results of the half-sine unit displacement excitation.
Direct Transient Analysis – Shock Wave

• Now create nodal force on the node of the tank mass.

\[ F_y = \frac{m}{D_A F} g_{\text{shock}} = \frac{141.3}{1.47} (32.2) = 3097 \text{ lbs} \]

Log-Log interpolation of spectrum at 108.1918 Hz:
\[ g_{\text{shock}} = 32.2 \]

• Update transient analysis to use this load set instead of enforced displacement.
• Run analysis.
Direct Transient Analysis Results

- Chart the Bush Y Force responses.
- Results comparison for vertical excitation:

<table>
<thead>
<tr>
<th>Direction</th>
<th>-Z End Reactions (lbs)</th>
<th>+Z End Reactions (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectrum Analysis</td>
<td>2378</td>
<td>2311</td>
</tr>
<tr>
<td>Transient Analysis</td>
<td>2295</td>
<td>2263</td>
</tr>
</tbody>
</table>

- Remember the second mode in vertical direction has a small amount of effective modal mass. This is captured in the spectral analysis but not the transient analysis.
Questions?

For questions on the material covered today, please contact David Cross.

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• Additional Resources:
  – Nastran Docs:
    • Basic Dynamics User’s Guide
    • Advanced Dynamics User’s Guide
  – Femap Help Topics:
    • Dynamic Analysis
    • Random Response of the Hinge Model
    • NASTRAN Random Response Analysis
    • Response Spectrum Analysis of Tower/Hinge Model
    • NASTRAN Response Spectrum Analysis
  – Please contact and I can provide a package of other reference documents that I have collected.
Special Note on Shock Spectrum Interpolation

- Shock Spectrum provided in log-log scale.
- Check boxes are visual only.
- Femap requires converting function to linear-linear scale, or manually editing the TABLED1 entry.