Getting Started with Composites Modeling and Analysis

IN THIS WEBINAR:

• Orthotropic materials and how to define them
• Composite Laminate properties and modeling
• Composite failure theories and postprocessing

PRESENTED BY:

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What is a Composite Material?

• Composite Material – a material made from two or more distinct materials with differing properties that are combined to produce a new material with unique properties
Lamina

- Lamina: A thin layer of composite material, usually containing unidirectional fibers or woven fibers in a fabric pattern. Also called a ply.
- Unidirectional plies have fibers in the Longitudinal (1) direction
- Fabric plies have fibers in both the Longitudinal (1) and Transverse (2)
Lamina

- Lamina: A thin layer of composite material, usually containing unidirectional fibers or woven fibers in a fabric pattern. Also called a ply.
Rule of Mixtures

• The stiffness of the final material will be defined by the Fiber Volume Fraction of the lamina

• Estimations of the material properties can be made, but experimental data should be used.

\[
E_1 = E_f V_f + E_m V_m \\
\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m} \\
\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m} \\
v_1 = v_f V_f + v_m V_m
\]
Orthotropic Material

- 3 planes of material symmetry
- 9 Engineering Constants
  - 3 elastic moduli
  - 3 shear moduli
  - 3 poisson ratio

\[
\begin{bmatrix}
\frac{1}{E_1} & -\nu_{21} & -\nu_{31} & 0 & 0 & 0 \\
-\nu_{12} & \frac{1}{E_2} & -\nu_{32} & 0 & 0 & 0 \\
-\nu_{13} & -\nu_{23} & \frac{1}{E_3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}}
\end{bmatrix}
\]
Laminate Stacking Sequence

- Plies are stacked together at different angles to create a Laminate

Source: http://www.composites.ugent.be/home_made_composites/what_are_composites.html
Composite Laminate

- A laminate is made of multiple lamina stacked together and held together by a Matrix.

- Terminology:
  - Balanced → equal number of + and – plies of the same angle
  - Symmetric → the plies in the laminate are a mirror image about the midplane
  - Quasi-Isotropic → Laminate has isotropic behavior in-plane
Classical Lamination Theory (CLT)

- Classical Lamination Theory (CLT)
  - CLT is the method used to calculate the ABD (stiffness) Matrix of the composite laminate
  - Each Lamina contains a “Reduced Stiffness Matrix”, $Q$

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} = \begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix} \begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix},
\]

\[
Q_{11} = \frac{E_1}{1 - \nu_{12} \nu_{21}}, \quad Q_{22} = \frac{E_2}{1 - \nu_{12} \nu_{21}},
\]

\[
Q_{12} = \frac{\nu_{12} E_2}{1 - \nu_{12} \nu_{21}}, \quad Q_{66} = G_{12}.
\]
Classical Lamination Theory (CLT)

• A transformation Matrix is defined to rotate stiffnesses from one coordinate system to the other

\[
[T] = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & 2 \sin \theta \cos \theta \\
\sin^2 \theta & \cos^2 \theta & -2 \sin \theta \cos \theta \\
-\sin \theta \cos \theta & \sin \theta \cos \theta & \left(\cos^2 \theta - \sin^2 \theta\right)
\end{bmatrix}, \quad [T]^{-1} = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & -2 \sin \theta \cos \theta \\
\sin^2 \theta & \cos^2 \theta & 2 \sin \theta \cos \theta \\
-\sin \theta \cos \theta & -\sin \theta \cos \theta & \left(\cos^2 \theta - \sin^2 \theta\right)
\end{bmatrix}
\]

• A new lamina stiffness matrix, denoted \([\overline{Q}]\), is defined:

\[
[\overline{Q}] = [T]^{-1} [Q] \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix} [T]
\]

• And the stresses and strains can be written in matrix form:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = \begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\
\bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}
\end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]
Classical Lamination Theory (CLT)

• The Laminate Stiffness Matrix, known as the ABD Matrix, is constructed by the following relations:

\[
A_{ij} = \sum_{k=1}^{n} [\bar{Q}_{ij}] (h_k - h_{k-1}) = \sum_{k=1}^{n} [\bar{Q}_{ij}^T] t_k
\]

\[
B_{ij} = \frac{1}{2} \sum_{k=1}^{n} [\bar{Q}_{ij}] (h_k^2 - h_{k-1}^2) = \sum_{k=1}^{n} [\bar{Q}_{ij}]_k \left( \frac{h_k + h_{k-1}}{2} \right)
\]

\[
D_{ij} = \frac{1}{3} \sum_{k=1}^{n} [\bar{Q}_{ij}] (h_k^3 - h_{k-1}^3) = \sum_{k=1}^{n} [\bar{Q}_{ij}]_k \left( \frac{t_k}{12} + \frac{t_k^2}{12} \right)
\]
FEMAP Demo
Laminate Offsets

Offset Bottom Surface = 0

Bottom of Laminate

Top of Laminate

\[ \vec{n} \]
Failure Theories

• Ply-by-ply failure theories predict a “Failure Index (FI)” for each ply
  – A failure index greater than or equal to 1.0 signifies a ply failure

• Examples of ply-by-ply failure theories:
  – Max Stress/Strain
  – Tsai-Hill
  – Tsai-Wu
  – Hoffman

Source: http://www.montana.edu/dcairns/documents/composites/The%20Tsai-Wu%20Failure%20Criterion.pdf
Hoffman Failure Theory

- The Hoffman Failure Criterion combines the stresses in a lamina (a single ply of a composite laminate) to predict failure
- A Failure Index is calculated and can be displayed
- Failure Index does not represent failure mode or percentage of failure

\[
\text{Failure Index} = \left( \frac{1}{X_t} - \frac{1}{X_c} \right) \sigma_1 + \left( \frac{1}{Y_t} - \frac{1}{Y_c} \right) \sigma_2 + \frac{\sigma_1^2}{X_t X_c} + \frac{\sigma_2^2}{Y_t Y_c} + \frac{\sigma_{12}^2}{S^2} - \frac{\sigma_1 \sigma_2}{X_t X_c}
\]

- Where \( X_t = \) tension allowable in “1” direction, \( X_c = \) compression
- Where \( Y_t = \) tension allowable in “2” direction, \( Y_c = \) compression
- \( S = \) Shear Allowable
  - \( \sigma_1 = \) applied stress in “1” direction
  - \( \sigma_2 = \) applied stress in “2” direction
  - \( \sigma_{12} = \) applied shear stress
Hoffman Failure Theory

Margins of Safety using the Hoffman Theory are calculated using:

\[ F_1 = \left( \frac{1}{X_t} - \frac{1}{X_c} \right); \quad F_2 = \left( \frac{1}{Y_t} - \frac{1}{Y_c} \right); \quad F_{11} = \frac{1}{X_tX_c}; \quad F_{22} = \frac{1}{Y_tY_c}; \quad F_{66} = \frac{1}{S^2}; \quad F_{12} = -\frac{F_{11}}{2} \]

\[ MS = \frac{2}{F_1\sigma_{11} + F_2\sigma_{22} + \sqrt{(F_1\sigma_{11} + F_2\sigma_{22})^2 + 4(F_{11}\sigma_{11}^2 + F_{22}\sigma_{22}^2 + F_{66}\tau_{12}^2 + 2F_{12}\sigma_{11}\sigma_{22})}} - 1.0 \]
Structural Design and Analysis (Structures.Aero)

**Structural Analysis**

- Team of stress engineers that help our clients design lightweight and load efficient structures.
- We service aerospace companies and other industries that require high level analysis.
- Specialty in composites and lightweight structures
- Tools used include hand analysis, HyperSizer, Femap, NX Nastran, Fibersim, NX, Solid Edge, Simcenter 3D, LS Dyna, and LMS.

**Software Sales and Support**

- Value added reseller providing software, training, and support for products we use on a daily basis.
- Support Femap, NX Nastran, Simcenter 3D, Fibersim, Solid Edge, and HyperSizer.
CAMX Tradeshow

• **December 12-14** at the Orange County Convention Center – Orlando, FL

• Largest event for composites and advanced materials

• SDA will be at **Booth U84**

• Want a free pass to walk the show? Email Marty Sivic at [msivic@structures.aero](mailto:msivic@structures.aero)
Questions?

For questions on the material covered today, please contact **Nick Mehlig**.

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For questions about pricing, or to see a demo, please contact **Marty Sivic**.

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